10 Model System Equilibration

10.1 Background and Concepts

SACSIM design has a cyclical relationship between network performance and trips: DAYSIM and the auxiliary trip models use network performance measures to model person-trips, which are then loaded to the network, determining congestion and network performance for the next iteration. The model system is in equilibrium when the network performance used as input to DAYSIM and the other trip models matches the network performance resulting from assignment of the resulting trips. Network performance for this purpose is times, distances, and costs measured zone-to-zone along the least-time paths (or more specifically, the paths of least generalized cost).

The theory of system equilibrium was developed based on trip-based models[13]. A wide range of trip-based models have a fixed point solution for all zone-to-zone and link flows, which can be solved with proper algorithms. These have been rare in practice until the 1990s, which saw development of many convergent model systems.

Almost all convergent trip-based models, at some stage in an iteration process, use the method of convex combinations. This is to update the current best solution of flows (zone-to-zone matrices and/or link volumes) with a weighted average of the previous best solution of those flows (x_{i-1}), and an alternative set of flows calculated by the new iteration shown in Equation 10-1. The first iteration normally uses network performance skim matrices based on free-flow link times.) When flows are combined in this manner, the result meets the same conservation-of-flow constraints as the iteration matrices.

Equation 10-1 SACSIM Iteration

$$(\mathbf{y}_i):\ \mathbf{x}_i = (1-\lambda)\mathbf{x}_{i-1} + \lambda\mathbf{y}_i$$

Where the step size λ must satisfy $0 < \lambda \le 1$. In the first iteration, there is no x_{i-1}, so λ must be 1.

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[13] Evans, Suzanne P. "Derivation and Analysis of Some Models for Combining Trip Distribution and Assignment", *Transportation Research*, Vol. 10, pp. 37-57 (1976).

Several trip-based model systems are defined so that the step size can be chosen at each iteration to optimize an objective function, or approach the solution to a variational inequality. But most models in practice do not satisfy those models' specific requirements, so the step size must be predetermined. The classic reliable workhorse is the Method of Successive Averages (MSA). This reliably converges for a wide range of models for which there is no determination of an iteration's optimal λ. This method chooses $\lambda = 1/i$, so that, in effect, after any iteration n, the solution approximation is the average of all the iteration-result vectors computed so far:

$$\mathbf{x}_i = \frac{\mathbf{y}_1 + \mathbf{y}_2 + ... + \mathbf{y}_i}{i}.$$ Some trip-based models converge reliably and more efficiently with a fixed step size[14], though care must be taken in the choice of that step size, which depends on the problem.

Equilibrium theory of trip-based models has unfortunately not been extended into activity-based models directly. In these, zone-to-zone flows are only an indirect result of more complex behavior models which cannot be reduced to the terms of the established equilibrium trip-based models. Activity models also have excessively vast choice sets to be able to split travel among all alternatives in proportion to their probability. Consequently, most, such as DAYSIM, are applied as Monte Carlo processes, randomly generating one outcome (household trip diary) per unit of analysis (household or person), and then aggregating the trips as zone-to-zone flows. Thus, the equilibration procedure employed by trip-based models can be applied by activity based model.

The equilibration procedure in SACSIM employs equilibrium assignment iteration loops (a-iterations) nested within iterations between the demand and assignment models (da-iterations).

Assignment is run for nine time periods, and each one employs multi-class equilibrium assignment, with classes composed of SOV, HOVs not using median HOV lanes, and HOVs using them. A convex combinations algorithm is used, with the step size α determined automatically by the Cube Voyager software, and closure criteria determined by the user: maximum number of iterations (N_i), and relative gap as defined by CUBE VOYAGER (g_i). Iterations stop when one of the closure criteria is satisfied.

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[14] Boyce, D., Ralevic-Dekic, B., and Bar-Gera, H., "Convergence of Traffic Assignments: How Much is Enough?" *Journal of Transportation Engineering,* 130:1(49), American Society of Civil Engineers, 2004.

There are several points in the model stream where it is possible to apply convex combinations as a "blending" of trips and/or volumes. The following are prevalent in the literature for convergent models:

(1) "Pre-assignment blending" - Blend the trip demand matrices from the system-iteration's demand model, with the previous system-iteration's blended trips, into a weighted average[15]. Then assign these new blended trips in equilibrium.

(2) "Post-assignment blending" - assign the new iteration trips alone in equilibrium, then blend those volumes with the previous system-iteration's blended link volumes[16].

(3) Assign each iteration's trips in an all-or-nothing assignment on the same paths used to derive the skims[17]. Most modeling software, and the several whole-matrix processes in SACSIM (and most trip-based models) conspire against the practicality of such an approach. Consequently, the Evans model and numerous generalizations[18] are rarely used in practice.

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[15] Boyce, David, et.al., "Introducing 'Feedback' into Four-Step Travel Forecasting Procedure vs. Equilibrium Solution of Combined Model", Transportation Research Record No. 1443, Transportation Research Board, Washington, D.C, 1994, pp. 65-74.

[16] Boyce, David, et.al. (1994), ibid.

[17] Evans (1976) ibid.

[18] Miller, Harvey J., "Towards Consistent Travel Demand Estimation in Transportation Planning: A Guide to the Theory and Practice of Equilibrium Travel Demand Modeling", U.S. Department of Transportation, Bureau of Transportation Statistics, June 27, 2001.